數學公式

一、總和(SUMMATION)

1. \( \sum_{i=1}^{n} X_i = X_1 + X_2 + X_3 + \cdots + X_n \)

2. \( \sum_{i=1}^{n} X_i^2 = X_1^2 + X_2^2 + X_3^2 + \cdots + X_n^2 \)

註：\( \sum_{i=1}^{n} X_i^2 \neq \left( \sum_{i=1}^{n} X_i \right)^2 \)

3. \( \sum_{i=1}^{n} X_i Y_i = X_1 Y_1 + X_2 Y_2 + X_3 Y_3 + \cdots + X_n Y_n \)

註：\( \sum_{i=1}^{n} X_i Y_i \neq \left( \sum_{i=1}^{n} X_i \right) \left( \sum_{i=1}^{n} Y_i \right) \)

公式：

1. \( \sum_{i=1}^{n} (X_i + Y_i) = \sum_{i=1}^{n} X_i + \sum_{i=1}^{n} Y_i \)

\( \sum_{i=1}^{n} (X_{1i} + X_{2i} + \cdots + X_{ki}) = \sum_{i=1}^{n} X_{1i} + \sum_{i=1}^{n} X_{2i} + \cdots + \sum_{i=1}^{n} X_{ki} \)

2. \( \sum_{i=1}^{n} (X_i - Y_i) = \sum_{i=1}^{n} X_i - \sum_{i=1}^{n} Y_i \)

3. \( \sum_{i=1}^{n} cX_i = c \sum_{i=1}^{n} X_i \)

\( \sum_{i=1}^{n} (c_1 X_{1i} + c_2 X_{2i} + \cdots + c_k X_{ki}) = c_1 \sum_{i=1}^{n} X_{1i} + c_2 \sum_{i=1}^{n} X_{2i} + \cdots + c_k \sum_{i=1}^{n} X_{ki} \)
例 1. \( \sum_{i=1}^{n} (X_i - \bar{X}) = \sum_{i=1}^{n} X_i - \sum_{i=1}^{n} \bar{X} = \sum_{i=1}^{n} X_i - n \bar{X} = n \bar{X} - n \bar{X} = 0 \)

例 2. \( \sum_{i=1}^{n} (X_i - \bar{X})^2 = \sum_{i=1}^{n} (X_i^2 - 2X_i \bar{X} + \bar{X}^2) \)

\[= \sum_{i=1}^{n} X_i^2 - 2 \bar{X} \sum_{i=1}^{n} X_i + \sum_{i=1}^{n} \bar{X}^2 \]

\[= \sum_{i=1}^{n} X_i^2 - 2 n \bar{X} \bar{X} + n \bar{X}^2 \]

\[= \sum_{i=1}^{n} X_i^2 - 2 n \bar{X}^2 + n \bar{X}^2 \]

\[= \sum_{i=1}^{n} X_i^2 - n \bar{X}^2 \]

二、微分基本公式:

1. \( \frac{dc}{dx} = 0 \) (c为常数) \( \frac{d}{dx} (ef(x)) = c \frac{df(x)}{dx} \)

2. \( \frac{dx^n}{dx} = nx^{n-1} \)

3. \( \frac{d}{dx} (f(x)g(x)) = \frac{df(x)}{dx} g(x) + f(x) \frac{dg(x)}{dx} \)

4. \( \frac{d}{dx} (f(x) \pm g(x)) = \frac{df(x)}{dx} \pm \frac{dg(x)}{dx} \)

5. \( \frac{d}{dx} \left( \frac{f(x)}{g(x)} \right) = \frac{\frac{df(x)}{dx} g(x) - \frac{dg(x)}{dx} f(x)}{[g(x)]^2} \)
三、對數

\[ \ln \text{是以自然數} e \approx 2.718282 \text{為底的對數符號，} \log \text{是以} 10 \text{為底的對數符號} \]

1. \( \ln(1) = 0, \quad \ln(e) = 1 \)

2. 若 \( a > 0, b > 0, r \in R \) 則
   a. \( \ln(e^a) = a \), \quad \text{If} \quad \ln(a) = b, \quad \text{then} \quad a = e^b \\
   b. \quad \ln(ab) = \ln a + \ln b \\
   c. \quad \ln(\frac{a}{b}) = \ln a - \ln b \\
   d. \quad \ln a^r = r \ln a \\

對數函數的導函數

1. \( \frac{d}{dx} \ln(f(x)) = \frac{f'(x)}{f(x)} \)

2. \( \frac{d}{dx} \ln x = \frac{1}{x}, \quad x > 0 \)

四、指數

1. \( e^a \cdot e^b = e^{a+b} \)

2. \( \frac{e^a}{e^b} = e^{a-b} \)

3. \( e^{\ln f(x)} = f(x), \quad e^{\ln x} = x \quad (x > 0, \quad f(x) > 0) \)

4. \( \ln(e^{f(x)}) = f(x), \quad \ln(e^x) = x \quad (x \in R, \quad f(x) \in R) \)

指數函數的導函數

1. \( \frac{d}{dx} e^x = e^x \)

2. \( \frac{d}{dx} a^x = a^x \cdot \ln a, \quad a > 0 \)

3. \( \frac{d}{dx} a^{f(x)} = a^{f(x)} \cdot f'(x) \cdot \ln a \)
五、積分簡易公式

1. \[ \int x^n dx = \frac{x^{n+1}}{n+1} + c, \quad n \neq -1 \]
2. \[ \int f^n(x) \cdot f'(x) dx = \frac{1}{n+1} f^{n+1}(x) + c, \quad n \neq -1 \]
3. \[ \int e^x dx = e^x + c \]
4. \[ \int a^x dx = \frac{1}{\ln a} a^x + c, \quad a > 0, a \neq 1 \]
5. \[ \int e^{f(x)} f'(x) dx = e^{f(x)} + c \]
6. \[ \int a^{f(x)} f'(x) (\ln a) dx = a^{f(x)} + c, \quad a > 0, a \neq 1 \]
7. \[ \int \frac{1}{x} dx = \ln |x| + c \]
8. \[ \int \frac{f'(x)}{f(x)} dx = \int \frac{1}{f(x)} df(x) = \ln |f(x)| + c \]

六、級數

1. 幂級數
   a. \[ \sum_{k=0}^{\infty} c_k x^k = c_0 + c_1 x + c_2 x^2 + \cdots \] 稱以 0 爲中心的冪級數。
   b. \[ \sum_{k=0}^{\infty} c_k (x-a)^k = c_0 + c_1 (x-a) + c_2 (x-a)^2 + \cdots \] 稱以 a 爲中心的冪級數。

2. 泰勒級數(Taylor’s Series)
   \[ f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!} (x-a)^k \]
   \[ = f(a) + \frac{f'(a)}{1!} (x-a) + \frac{f''(a)}{2!} (x-a)^2 + \cdots + \frac{f^{(k)}(a)}{k!} (x-a)^k + \cdots \]
   稱 f(x) 在 x = a 處之泰勒級數
3. 麥克勞林級數(Maclaurin’s Series)

\[
f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(0)}{n!} x^k
\]

\[= f(0) + \frac{f'(0)}{1!} x + \frac{f''(0)}{2!} x^2 + \cdots + \frac{f^{(k)}(0)}{n!} x^k + \cdots
\]

稱 \( f(x) \) 在 \( x = 0 \) 處之泰勒級數，一般稱為麥克勞林級數。

4. 基本函數的麥克勞林級數

a. \( \frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots = \sum_{k=0}^{\infty} x^k \), \(|x| < 1\)

b. \( \frac{1}{1+x} = 1 - x + x^2 - x^3 + \cdots = \sum_{k=0}^{\infty} (-1)^k x^k \), \(|x| < 1\)

c. \( e^x = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^k}{k!} + \cdots = \sum_{k=0}^{\infty} \frac{x^k}{k!} \), \( x \in R \)